Curtin University, Semester 2, 2026

ECOM5005
Tutorial 4 Detailed Solutions

Elementary Probability - classroom-ready worked solutions

Learning Outcomes

1. Recognise and understand basic probability concepts.
2. Calculate probabilities of simple, marginal and joint events.
3. Understand and calculate conditional probabilities and determine whether events are independent or not.
4. To be able to use Bayes' Theorem to revise probabilities.
5. Use counting rules to calculate the number of possible outcomes.

PART 1 - Detailed worked solutions

Question 1. The two-child problem

Question

For a two-child family with equally likely ordered outcomes S = {(G,G), (G,B), (B,G), (B,B)}:

(a) Find P(both girls | first child is a girl).

(b) Find P(both girls | at least one child is a girl).

Detailed Solution

Define A = both children are girls = {(G,G)}.

Define B = first child is a girl = {(G,G), (G,B)}.

Define C = at least one child is a girl = {(G,G), (G,B), (B,G)}.

Because all four ordered outcomes are equally likely, P(A)=1/4, P(B)=2/4=1/2 and P(C)=3/4.

Part (a)

$$P(A \mid B) = P(A \cap B) / P(B) = P(A) / P(B) = (1/4) / (1/2) = 1/2$$

Once we are told that the first child is a girl, only two outcomes remain possible: (G,G) and (G,B). Exactly one of these has two girls, so the probability is 1/2.

Part (b)

$$P(A \mid C) = P(A \cap C) / P(C) = P(A) / P(C) = (1/4) / (3/4) = 1/3$$

Knowing only that at least one child is a girl removes (B,B), leaving three equally likely ordered outcomes: (G,G), (G,B) and (B,G). Only one has two girls.

Answer Summary

(a) 1/2 = 50%.

(b) 1/3 ≈ 33.33%.

The difference occurs because the conditioning information changes the relevant sample space.

Question 2. Two fair coins and one two-headed coin

Question

A box contains two regular coins and one two-headed coin. A coin is selected at random and tossed. Find the probability of heads.

Let C_1 = regular coin and C_2 = two-headed coin. Then $P(C_1)=2/3$ and $P(C_2)=1/3$. Also $P(H|C_1)=1/2$ and $P(H|C_2)=1$.

$$P(H) = P(H \mid C_1)P(C_1) + P(H \mid C_2)P(C_2)$$

$$= (1/2)(2/3) + (1)(1/3) = 1/3 + 1/3 = 2/3$$

A useful intuitive check is to imagine the six equally likely "coin-face" possibilities created by choosing among the three coins and then observing a face. Four of the six lead to heads, giving 4/6 = 2/3.

Answer Summary

P(H) = 2/3 ≈ 66.67%.

Question 3. Spam email and the word "offer"

Question

'Offer' appears in 80% of spam emails and 10% of desired emails. Suppose 30% of received emails are spam. Given that a new email contains 'offer', find the probability that it is spam.

Let S = spam and O = contains the word "offer". The given probabilities are P(S)=0.30, P(S′)=0.70, P(O|S)=0.80 and P(O|S′)=0.10.

Method 1 - Expected frequencies out of 100 emails

Email type	Total	Contains "offer"	Does not contain "offer"
Spam	30	24	6
Desired	70	7	63
Total	100	31	69

$$P(S \mid O) = 24 / 31 \approx 0.7742 = 77.42\%$$

Method 2 - Bayes' Theorem

$$P(S \mid O) = [P(O \mid S)P(S)] / [P(O \mid S)P(S) + P(O \mid S')P(S')]$$

$$= (0.80\times0.30) / [(0.80\times0.30) + (0.10\times0.70)] = 0.24 / 0.31 \approx 0.7742$$

Answer Summary

Given that the email contains "offer", the probability that it is spam is approximately 77.42%.

Question 4. E-commerce fraud screening

Question

P(Fraud)=3%. The screening system flags 90% of fraudulent transactions and 5% of legitimate transactions.

Construct an expected-frequency table for 10,000 transactions, then find P(flagged), P(fraud | flagged), P(fraud | not flagged), and interpret the result.

Step 1 - Convert the probabilities into expected counts

Out of 10,000 transactions, 3% are expected to be fraudulent: 10,000×0.03=300. The remaining 9,700 are legitimate.

Among the 300 fraudulent transactions, 90% are flagged: 300×0.90=270, leaving 30 not flagged. Among the 9,700 legitimate transactions, 5% are flagged: 9,700×0.05=485, leaving 9,215 not flagged.

Transaction type	Flagged	Not flagged	Total
Fraudulent	270	30	300
Legitimate	485	9215	9700
Total	755	9245	10000

Step 2 - Probability a transaction is flagged

$$P(A) = 755 / 10,000 = 0.0755 = 7.55\%$$

Step 3 - Probability of fraud given that the transaction is flagged

$$P(F \mid A) = 270 / 755 \approx 0.3576 = 35.76\%$$

Step 4 - Probability of fraud given that the transaction is not flagged

$$P(F \mid A') = 30 / 9,245 \approx 0.003245 = 0.32\%$$

The screening system has high sensitivity to fraud, but fraud is rare in the overall population. Consequently, many flagged transactions are legitimate false positives. This is the base-rate effect: P(flagged | fraud) is not the same quantity as P(fraud | flagged).

Answer Summary

P(flagged)=7.55%.

P(fraud | flagged)=35.76%.

P(fraud | not flagged)=0.32%.

A 90% fraud-detection rate does not imply that 90% of flagged transactions are fraudulent because legitimate transactions are far more common and some of them are also flagged.

Question 5. Retail campaign design and counting rules

Question

Eight candidate products are available: 3 premium and 5 standard. Find ordered homepage arrangements, unordered bundles, and probabilities involving premium products.

Part (a) - Ordered homepage positions

Hero, Secondary and Tertiary are distinct positions, so order matters. We use a permutation.

$$P(8,3) = 8! / (8-3)! = 8 \times 7 \times 6 = 336$$

Part (b) - Unordered three-product bundle

A bundle contains three products but has no ranking, so order does not matter. We use a combination.

$$C(8,3) = 8! / [3!5!] = 56$$

Part (c) - Exactly two premium products

Choose 2 of the 3 premium products and 1 of the 5 standard products.

$$P(\text{exactly 2 premium}) = [C(3,2)C(5,1)] / C(8,3) = (3 \times 5)/56 = 15/56 \approx 26.79\%$$

Part (d) - At least one premium product

Use the complement: “at least one premium” is the complement of “all three are standard”.

$$P(\text{at least 1 premium}) = 1 - C(5,3)/C(8,3) = 1 - 10/56 = 46/56 \approx 82.14\%$$

Part (e): A permutation is appropriate when position/order changes the outcome; a combination is appropriate when only the selected set matters.

Answer Summary

(a) 336 ordered homepage displays.

(b) 56 unordered bundles.

(c) 26.79% contain exactly two premium products.

(d) 82.14% contain at least one premium product.

PART 2 - Data exercise: super-detailed Excel solution

Question 6. Australian labour-force probability analysis

What the source workbook contains

The UE worksheet contains a table for the Australian civilian population aged 15 years and over in May 2017. Values are reported in thousands.

The five status categories are full-time employment, part-time employment, two unemployment categories, and not in the labour force. Male, Female and Total columns are provided.

Common mistakes to avoid

- Do not divide by the overall population when the question says "given male" or "given full-time employed". Conditional probabilities use the conditioned group as the denominator.
- Do not confuse "not employed" with "unemployed". In this exercise, not employed includes both unemployed categories plus not in the labour force.
- The unemployment-rate denominator is the labour force, not the total civilian population.
- The participation-rate denominator is the total civilian population.
- Keep the raw UE worksheet unchanged. Perform all calculations on a separate Analysis worksheet.

Step 0 - Open the workbook and create a safe working sheet

1. Open UE_Australia.xlsx in Microsoft Excel.
2. At the bottom of the workbook, click the plus (+) button to insert a new worksheet.
3. Double-click the new worksheet tab and rename it Analysis.
4. Return to the UE sheet. The source table occupies columns C:F, with the headings on row 4 and data on rows 5:10.
5. Select C4:F10, press Ctrl+C, return to Analysis, click A3, and paste. This gives you a clean working copy beginning in A3:D9.
6. Do not change the original UE sheet. This makes it easy to check your work if a formula is accidentally overwritten.

Step 1 - Confirm the working table

Labour force status	Male	Female	Total ('000)
Employed full-time	5,296.0	3,001.2	8,297.2
Employed part-time	1,230.5	2,678.2	3,908.7
Unemployed and looking for full-time work	277.6	205.4	483.0
Unemployed and not looking for full-time work	90.3	130.1	220.4
Not in labour force	2,859.4	4,060.5	6,919.9
Total civilian population aged 15 years and over	9,753.8	10,075.4	19,829.2

Check: Male total + Female total = 9,753.8 + 10,075.4 = 19,829.2, which matches the overall total. Because every value is in thousands, the scale cancels when we form probabilities and rates.

Step 2 - Build the derived-measures table

In Analysis!A12:D19, create the table shown below. Type the labels, then enter the formulas exactly as shown. Excel formulas should begin with an equals sign.

Row	Measure	Male formula	Female formula	Overall formula
13	Population	=B9	=C9	=D9
14	Employed	=SUM(B4:B5)	=SUM(C4:C5)	=SUM(D4:D5)
15	Unemployed	=SUM(B6:B7)	=SUM(C6:C7)	=SUM(D6:D7)
16	Labour force	=B14+B15	=C14+C15	=D14+D15
17	Not employed	=B15+B8	=C15+C8	=D15+D8
18	Unemployment rate	=B15/B16	=C15/C16	=D15/D16
19	Participation rate	=B16/B13	=C16/C13	=D16/D13

After entering the formulas in row 18 and row 19, select B18:D19, open Home > Number, and choose Percentage. Increase decimal places until two decimal places are displayed.

Final derived-measures table

Derived measure	Male	Female	Overall
Population	9,753.8	10,075.4	19,829.2
Employed	6,526.5	5,679.4	12,205.9
Unemployed	367.9	335.5	703.4
Labour force	6,894.4	6,014.9	12,909.3
Not employed	3,227.3	4,396.0	7,623.3
Unemployment rate	5.34%	5.58%	5.45%
Participation rate	70.68%	59.70%	65.10%

Step 3 - Part (a): probability that a randomly selected person is female

The denominator is the total civilian population and the numerator is the total female population.

$$P(\text{Female}) = 10{,}075.4 \,/\, 19{,}829.2 = 0.508109 \approx 50.81\%$$

Excel formula (for the Analysis layout above): =C9/D9

Step 4 - Part (b): probability that a randomly selected male is not employed

The phrase “randomly selected male” means this is conditional on being male. The denominator must therefore be the male population, not the total population. “Not employed” equals unemployed plus not in labour force.

$$\text{Male not employed} = 277.6 + 90.3 + 2{,}859.4 = 3{,}227.3$$

$$P(\text{Not employed} \mid \text{Male}) = 3{,}227.3 \,/\, 9{,}753.8 = 0.330876 \approx 33.09\%$$

Excel formula: =B17/B13

Step 5 - Part (c): probability of female given full-time employment

Because the person is known to be employed full time, the full-time total becomes the denominator.

$$P(\text{Female} \mid \text{Full-time}) = 3{,}001.2 \,/\, 8{,}297.2 = 0.361712 \approx 36.17\%$$

Excel formula: =C4/D4

Step 6 - Part (d): test statistical independence

Method 1 uses the conditional-probability definition of independence. If Female and Full-time were independent, then knowing that a person is full-time employed would not change the probability that the person is female.

$$P(\text{Female}) = 50.81\% \quad \text{versus} \quad P(\text{Female} \mid \text{Full-time}) = 36.17\%$$

These are not equal, so the events are not independent.

Method 2 provides a cross-check. Under independence, P(Female ∩ Full-time) must equal P(Female)×P(Full-time).

$$P(\text{Female} \cap \text{Full-time}) = 3{,}001.2 \,/\, 19{,}829.2 = 15.14\%$$

$$P(\text{Female})P(\text{Full-time}) = 50.81\% \times 41.84\% = 21.26\%$$

Again the values are clearly different. Therefore, sex and full-time-employment status are statistically associated in this table.

Step 7 - Part (e): joint probability of male and full-time employment

$$P(\text{Male} \cap \text{Full-time}) = 5{,}296.0 \,/\, 19{,}829.2 = 0.267081 \approx 26.71\%$$

Excel formula: =B4/D9

Step 8 - Part (f): unemployment rates

For each group, the unemployment rate is Unemployed / Labour force. The labour force consists of employed people plus unemployed people.

$$\text{Male unemployment rate} = 367.9 \,/\, 6{,}894.4 = 5.34\%$$

$$\text{Female unemployment rate} = 335.5 \,/\, 6{,}014.9 = 5.58\%$$

$$\text{Overall unemployment rate} = 703.4 \,/\, 12{,}909.3 = 5.45\%$$

Excel formulas: =B15/B16, =C15/C16 and =D15/D16

Step 9 - Part (g): participation rates

For each group, participation rate is Labour force / Total civilian population.

$$\text{Male participation rate} = 6{,}894.4 \,/\, 9{,}753.8 = 70.68\%$$

$$\text{Female participation rate} = 6{,}014.9 \,/\, 10{,}075.4 = 59.70\%$$

$$\text{Overall participation rate} = 12{,}909.3 \,/\, 19{,}829.2 = 65.10\%$$

Excel formulas: =B16/B13, =C16/C13 and =D16/D13

Step 10 - Build a probability-results table

Create a compact summary table in columns F:G. This makes checking easier and gives you a clear final output for the tutorial.

Probability / test	Final result
P(Female)	50.81%
P(Not employed | Male)	33.09%
P(Female | Full-time)	36.17%
P(Female ∩ Full-time)	15.14%
P(Female) × P(Full-time)	21.26%
P(Male ∩ Full-time)	26.71%

Independence conclusion

Female and full-time employment are NOT statistically independent. Both checks fail: P(Female | Full-time) ≠ P(Female), and P(Female ∩ Full-time) ≠ P(Female)P(Full-time).

Step 11 - Part (h): create the labour-force-status chart in Excel

1. Select A3:C8 on the Analysis sheet. This includes the status labels plus Male and Female values, but excludes the total row.
2. Go to Insert > Column or Bar Chart > Clustered Column.
3. Click the chart title and type: Labour Force Status by Sex - Australia, May 2017.
4. Make sure the legend shows Male and Female.
5. If Excel places categories/series incorrectly, use Chart Design > Switch Row/Column.
6. Resize the chart so all category labels can be read. Long labels may wrap automatically.

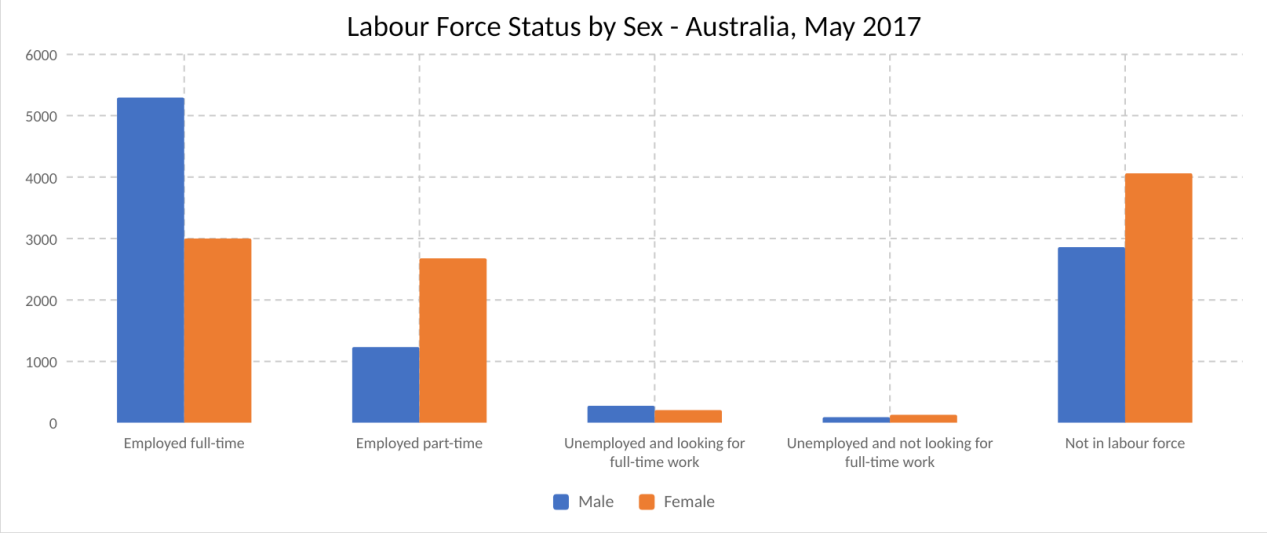

Figure 1: Final clustered column chart for labour-force status by sex

Step 12 - Part (i): create the rates comparison chart in Excel

1. Create a small chart table with rows Unemployment and Participation and columns Male, Female and Overall. Link the cells to the rates you calculated in Step 2.
2. Select the entire small table, including row and column headings.
3. Go to Insert > Column or Bar Chart > Clustered Column.
4. Set the chart title to: Unemployment and Participation Rates.
5. Right-click the vertical axis, choose Format Axis, and set Number to Percentage with 0 or 1 decimal place.
6. Check that the participation bars are around 60%-71% and the unemployment bars are around 5%-6%. If you see values near 500% or 700%, your percentage formatting or formulas are incorrect.

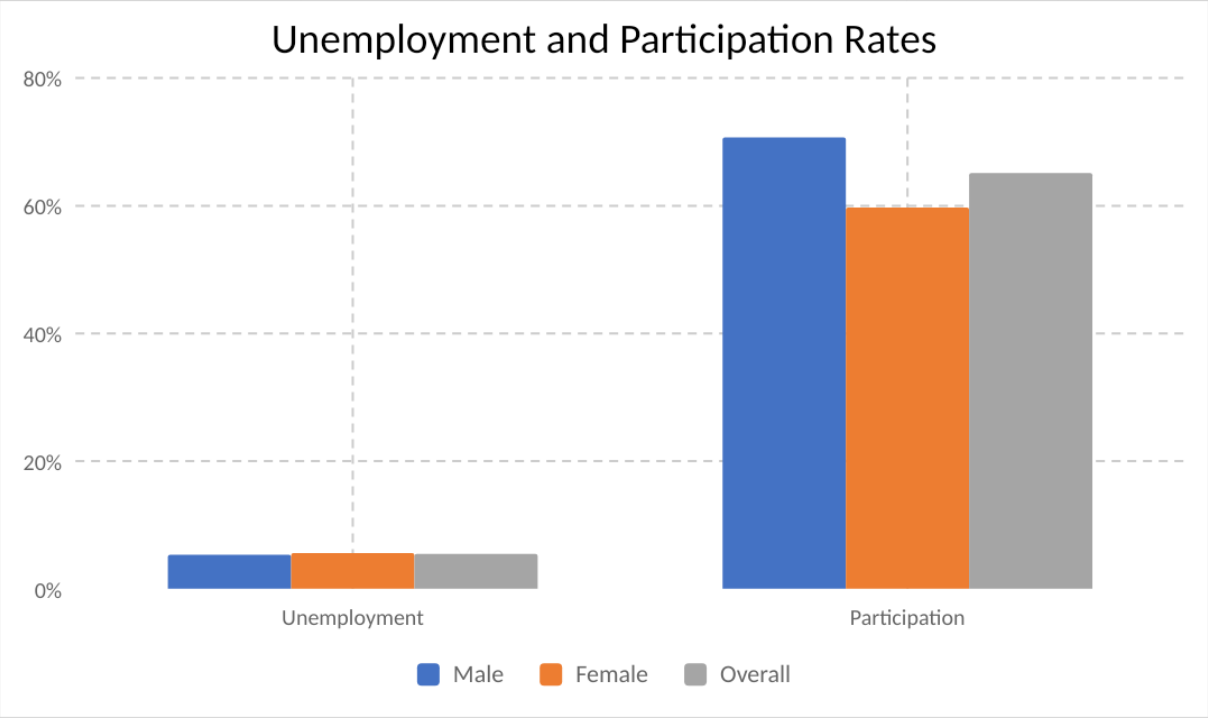

Figure 2: Final comparison of unemployment and participation rates

Step 13 - Part (j): interpretation

A strong answer does more than repeat numbers. It links the probability or rate to a substantive labour-market observation. For example:

- Females make up slightly more than half of the civilian population aged 15 years and over in this table (50.81%).
- Among full-time employed people, only 36.17% are female, which is substantially below the overall female share of 50.81%; this is consistent with the formal conclusion that Female and Full-time employment are not independent.
- The male and female unemployment rates are similar (5.34% and 5.58%), but the participation rate is notably higher for males (70.68%) than females (59.70%) in the May 2017 data.

Final answer table for Part 2

Item	Answer
(a) P(Female)	50.81%
(b) P(Not employed | Male)	33.09%
(c) P(Female | Full-time)	36.17%
(d) Independence?	No - not independent
(e) P(Male ∩ Full-time)	26.71%
(f) Unemployment rate - Male	5.34%
(f) Unemployment rate - Female	5.58%
(f) Unemployment rate - Overall	5.45%
(g) Participation rate - Male	70.68%
(g) Participation rate - Female	59.70%
(g) Participation rate - Overall	65.10%

Excel formula map - quick reference

P(Female): =C9/D9
P(Not employed | Male): =B17/B13
P(Female | Full-time): =C4/D4
P(Female ∩ Full-time): =C4/D9
P(Female)×P(Full-time): =(C9/D9)*(D4/D9)
P(Male ∩ Full-time): =B4/D9
Unemployment rates: =B15/B16, =C15/C16, =D15/D16
Participation rates: =B16/B13, =C16/C13, =D16/D13

PART 3 - Pearson MyLab

Please logon to MyLab and complete the following tasks.

1. Complete Quiz 2 which has been released on this Tuesday (25-Aug-2026) and will be due on next Tuesday (1-Sep-2026) at 5:00pm AWST.

Note: If you are having issues with obtaining access to MyLab, please re-read the instructions available in the welcome topic. If you are still unsuccessful, contact the teaching team explaining the issue. It is important that MyLab tasks are completed on a weekly basis.